Algebra and graphs C2.1

- **Constant: fixed number, $1, 2, \ldots$**

- **Variable: any number, $x, y, \ldots$**

- **Term: a product, $2, x, 3y, \ldots$**

- **Expression: a sum, $x + 1, 2x - 3y, \ldots$**

- **Equation: equal expressions, $x + 1 = 2, \ldots$**

- **Identity: true for all $x, x + x = 2x, \ldots$**

- **Formula: relationship of quantities, $A = b \times h$**

Algebra and graphs C2.1

- **Substitute: replace the variable with its given value:**

  $5x^2 - 4x + 7$

  when $x = -2$:

  $5(-2)^2 - 4(-2) + 7$

  $= 5 \times 4 + 8 + 7$

  $= 35$

- **Rearrange: use inverse operations to both sides of the equation or formula:**

  Make L the subject of the formula:

  $$\frac{T}{2\pi} = \sqrt{\frac{L}{g}} \qquad (\;^2)$$

  $$\left(\frac{T}{2\pi}\right)^2 = \frac{L}{g} \qquad (\times g)$$

  $$g\left(\frac{T}{2\pi}\right)^2 = L$$

- **Construct: translate into mathematical operations:**

  I'm thinking of a number. When 3 more than half of my number is doubled, I get 12:

  $$2\left(\frac{x}{2} + 3\right) = 12.$$

  What's my number?

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Expanding brackets: multiplication

$3x(2x - 5y) = 6x^2 - 15xy$

$\times$	$2x$	$-5y$
$3x$	$6x^2$	$-15xy$

$(3x + 1)(2x - 5) = 6x^2 - 15x + 2x - 5$

$= 6x^2 - 13x - 5$

$\times$	$2x$	-5
$3x$	$6x^2$	$-15x$
$+1$	$+2x$	-5

$(3x + 1)(3x - 1) = 9x^2 - 3x + 3x - 1$

$= 9x^2 - 1$

$\times$	$3x$	-1
$3x$	$9x^2$	$-3x$
$+1$	$+3x$	-1

Extract common factors: division

- **Factorising binomials:**

$$6x^2 - 15xy = 3x(2x - 5y)$$

$\times$	$2x$	$-5y$
$3x$	$6x^2$	$-15xy$

- **Factorising trinomials:**

$$6x^2 - 13x - 5 = (3x + 1)(2x - 5)$$

$\times$	$2x$	-5
$3x$	$6x^2$	$-15x$
$+1$	$+2x$	-5

- **Factorising the difference of two squares:**

$$9x^2 - 1 = 9x^2 + 0x - 1 = (3x + 1)(3x - 1)$$

$\times$	$3x$	-1
$3x$	$9x^2$	$-3x$
$+1$	$+3x$	-1

Index laws:

- $x^1 = x$

- $x^0 = 1 \ (x \neq 0)$

- $x^{-n} = \frac{1}{x^n}$

- $x^m \times x^n = x^{m+n}$

- $(x^m)^n = x^{mn}$

- $x^{\frac{m}{n}} = \sqrt[n]{x^m}$

- $x^m \div x^n = x^{m-n}$

- $3x^4 \times 5x = 3 \times 5 \times x^4 \times x^1 = 15x^5$

- $\frac{10x^3}{2x^2} = \frac{10}{2} \times \frac{x^3}{x^2} = 5 \times x^1 = 5x$
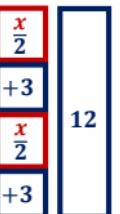
- $(3x^4)^3 = (3)^3 \times (x^4)^3 = 27x^{12}$

Algebra and graphs C2.5
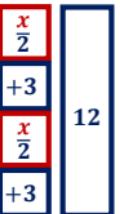
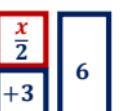**Linear equations: use simplifying or inverse operations to both sides of the equation:**

$2\left(\dfrac{x}{2} + 3\right) = 12$ $(\div 2)$

$2\left(\dfrac{x}{2} + 3\right) = 12$ $(expand)$
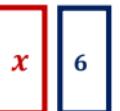
$\dfrac{x}{2} + 3 = 6$ (-3)

$x + 6 = 12$ (-6)

$\dfrac{x}{2} = 3$ $(\times 2)$

$x = 6$

$x = 6$
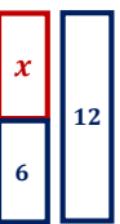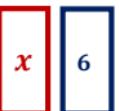
Simultaneous equations: eliminating variables

① $2x + y = 10$
② $x + 2y = 8$ $\quad (\times 2)$
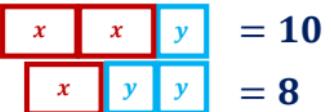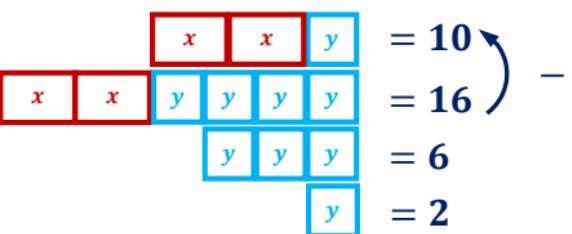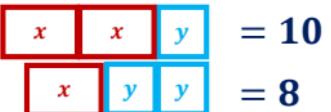
| x | x | y | $= 10$ |
| x | y | y | $= 8$ |

① $2x + y = 10$
② $2x + 4y = 16$ $\quad \Big) -$
$ 3y = 6$
$y = 2$

① $2x + y = 10$ $\quad (\times 2)$
② $x + 2y = 8$

① $4x + 2y = 20$ $\quad \Big) -$
② $x + 2y = 8$
$3x = 12$
$x = 4$
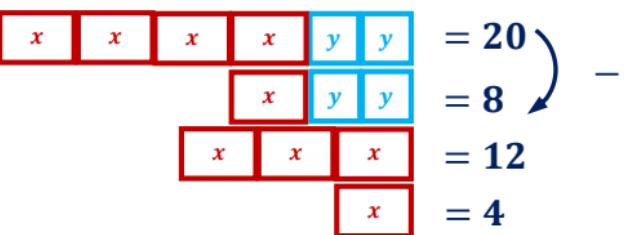
Linear sequences:

n^{th} term of $2, 5, 8, 11 \ldots$

n	0	1	2	3	4
Term	-1	2	5	8	11
$3n$	0	3	6	9	12

n^{th} term $= 3n - 1 = -1 + 3n$

$$y = 3n - 1$$

n^{th} term of $18, 13, 8, 3 \ldots$

n	0	1	2	3	4
Term	23	18	13	8	3
$-5n$	0	-5	-10	-15	-20

n^{th} term $= -5n + 23 = 23 - 5n$

$$y = -5n + 23$$

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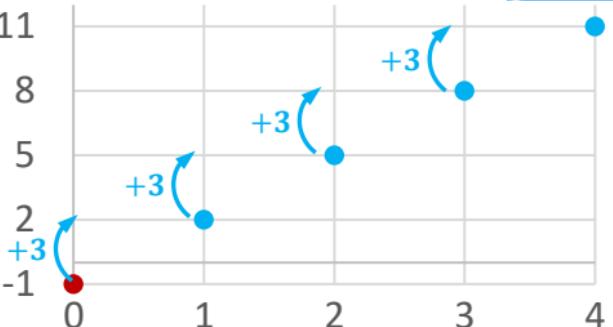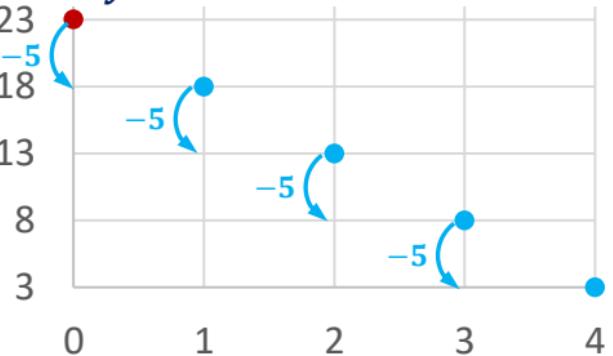
Algebra and graphs C2.10

Travel graphs:

$Speed\ 1: 500m/2min = 250m/min$

$Speed\ 2: 0m/5min\ (stationary)$

$Speed\ 3: 500m/5min = 100m/min$

$Total\ speed: \dfrac{1000m}{12min} = 83\dfrac{1}{3}m/min$
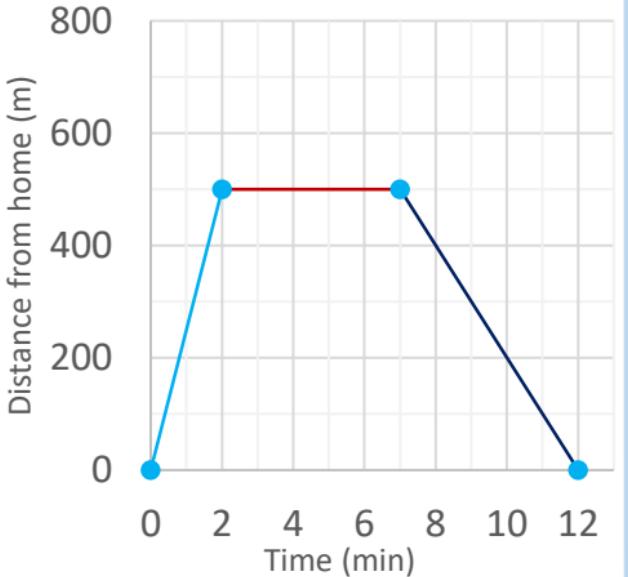
Conversion graphs:

$Rate: \$1.31\ per\ £1$

$£1 = \$1.31$

$£8 = \$10.48$

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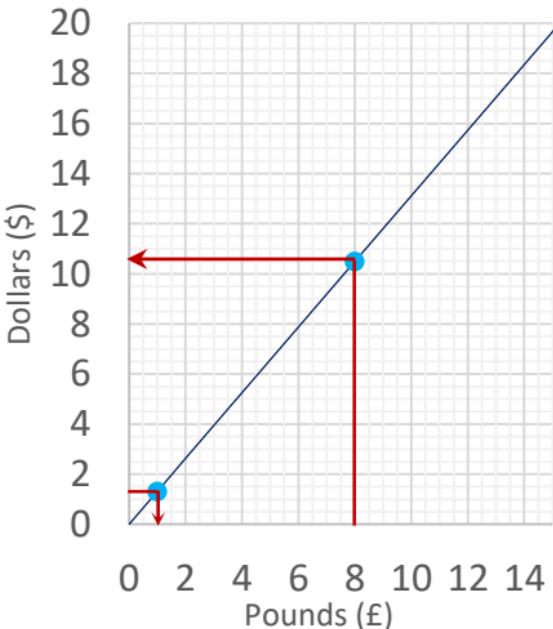
$y = 3x - 1 = 3\left(x - \frac{1}{3}\right)$

x	-4	-2	0	2	4
y	-13	-7	-1	5	11

$y = 0$ at $x = \frac{1}{3}$

$y = x^2 - x - 2 = (x - 2)(x + 1)$

x	-4	-2	0	2	4
y	18	4	-2	0	10

$y = 0$ at $x = -1, x = 2$

Estimate from graph:

$x^2 - x - 2 = 3x - 1 \quad \{-4 \leq x \leq 4\}$

$x \approx -0.24, y \approx -1.7$ at ✗