

# Algebra and graphs C2.1



- **Constant:** fixed number,  $1, 2, \dots$
- **Variable:** any number,  $x, y, \dots$
- **Term:** a product,  $2, x, 3y, \dots$
- **Expression:** a sum,  $x + 1, 2x - 3y, \dots$
- **Equation:** equal expressions,  $x + 1 = 2, \dots$
- **Identity:** true for all  $x, x + x = 2x, \dots$
- **Formula:** relationship of quantities,  $A = b \times h$

# Algebra and graphs C2.1



- **Substitute:** replace the variable with its given value:

$$5x^2 - 4x + 7$$

when  $x = -2$ :

$$5(-2)^2 - 4(-2) + 7$$

$$= 5 \times 4 + 8 + 7$$

$$= 35$$

- **Rearrange:** use inverse operations to both sides of the equation or formula:

*Make  $L$  the subject of the formula:*

$$\frac{T}{2\pi} = \sqrt{\frac{L}{g}} \quad (\text{ } ^2)$$

$$\left(\frac{T}{2\pi}\right)^2 = \frac{L}{g} \quad (\times g)$$

$$g\left(\frac{T}{2\pi}\right)^2 = L$$

- **Construct:** translate into mathematical operations:

*I'm thinking of a **number**. When 3 more than half of my number is doubled, I get 12:*

$$2\left(\frac{x}{2} + 3\right) = 12.$$

*What's my number?*

## Algebra and graphs C2.2



### Expanding brackets: multiplication

$$3x(2x - 5y) = 6x^2 - 15xy$$

*Note: In the original image, a blue arc with an 'x' above it connects the '3x' and '2x' terms, and another blue arc connects the '3x' and '5y' terms.*

	×	2x	-5y
3x		6x <sup>2</sup>	-15xy

$$(3x + 1)(2x - 5) = 6x^2 - 15x + 2x - 5$$
$$= 6x^2 - 13x - 5$$

	×	2x	-5
3x		6x <sup>2</sup>	-15x
+1		+2x	-5

*Note: In the original image, a red oval highlights the terms 6x^2, -15x, +2x, and -5.*

$$(3x + 1)(3x - 1) = 9x^2 - 3x + 3x - 1$$
$$= 9x^2 - 1$$

	×	3x	-1
3x		9x <sup>2</sup>	-3x
+1		+3x	-1

*Note: In the original image, a red oval highlights the terms 9x^2, -3x, +3x, and -1.*

## Algebra and graphs C2.2



### Extract common factors: division

#### • Factorising binomials:

$$6x^2 - 15xy = 3x(2x - 5y)$$

	×	2x	-5y
3x		6x <sup>2</sup>	-15xy

#### • Factorising trinomials:

$$6x^2 - 13x - 5 = (3x + 1)(2x - 5)$$

	×	2x	-5
3x		6x <sup>2</sup>	-15x
+1		+2x	-5

#### • Factorising the difference of two squares:

$$9x^2 - 1 = 9x^2 + 0x - 1 = (3x + 1)(3x - 1)$$

	×	3x	-1
3x		9x <sup>2</sup>	-3x
+1		+3x	-1



## Index laws:

- $x^1 = x$
- $x^0 = 1$  ( $x \neq 0$ )
- $x^{-n} = \frac{1}{x^n}$
- $x^m \times x^n = x^{m+n}$
- $(x^m)^n = x^{mn}$
- $x^{\frac{m}{n}} = \sqrt[n]{x^m}$
- $x^m \div x^n = x^{m-n}$
- $3x^4 \times 5x = 3 \times 5 \times x^4 \times x^1 = 15x^5$
- $\frac{10x^3}{2x^2} = \frac{10}{2} \times \frac{x^3}{x^2} = 5 \times x^1 = 5x$
- $(3x^4)^3 = (3)^3 \times (x^4)^3 = 27x^{12}$

# Algebra and graphs C2.5



Linear equations: use simplifying or inverse operations  
to both sides of the equation:

$$2\left(\frac{x}{2} + 3\right) = 12 \quad (\div 2)$$

$\frac{x}{2}$	12
+3	
$\frac{x}{2}$	
+3	

$$2\left(\frac{x}{2} + 3\right) = 12$$

*(expand)*

$\frac{x}{2}$	12
+3	
$\frac{x}{2}$	
+3	

$$\frac{x}{2} + 3 = 6 \quad (-3)$$

$\frac{x}{2}$	6
+3	

$$x + 6 = 12 \quad (-6)$$

$x$	12
6	

$$\frac{x}{2} = 3 \quad (\times 2)$$

$\frac{x}{2}$	3
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$$x = 6$$

$x$	6
-----	---

$$x = 6$$

$x$	6
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# Algebra and graphs C2.5



## Simultaneous equations: eliminating variables

$$\textcircled{1} \quad 2x + y = 10$$

$$\textcircled{2} \quad x + 2y = 8 \quad (\times 2)$$

$$\begin{array}{|c|c|c|} \hline x & x & y \\ \hline \end{array} = 10$$

$$\begin{array}{|c|c|c|} \hline x & y & y \\ \hline \end{array} = 8$$

$$\textcircled{1} \quad 2x + y = 10$$

$$\textcircled{2} \quad 2x + 4y = 16 \quad -$$

$$3y = 6$$

$$y = 2$$

$$\begin{array}{|c|c|c|} \hline x & x & y \\ \hline \end{array} = 10$$

$$\begin{array}{|c|c|c|c|c|c|} \hline x & x & y & y & y & y \\ \hline \end{array} = 16 \quad -$$

$$\begin{array}{|c|c|c|} \hline y & y & y \\ \hline \end{array} = 6$$

$$\begin{array}{|c|} \hline y \\ \hline \end{array} = 2$$

$$\textcircled{1} \quad 2x + y = 10 \quad (\times 2)$$

$$\textcircled{2} \quad x + 2y = 8$$

$$\begin{array}{|c|c|c|} \hline x & x & y \\ \hline \end{array} = 10$$

$$\begin{array}{|c|c|c|} \hline x & y & y \\ \hline \end{array} = 8$$

$$\textcircled{1} \quad 4x + 2y = 20$$

$$\textcircled{2} \quad x + 2y = 8 \quad -$$

$$3x = 12$$

$$x = 4$$

$$\begin{array}{|c|c|c|c|c|c|} \hline x & x & x & x & y & y \\ \hline \end{array} = 20$$

$$\begin{array}{|c|c|c|} \hline x & y & y \\ \hline \end{array} = 8 \quad -$$

$$\begin{array}{|c|c|c|} \hline x & x & x \\ \hline \end{array} = 12$$

$$\begin{array}{|c|} \hline x \\ \hline \end{array} = 4$$

# Algebra and graphs C2.7



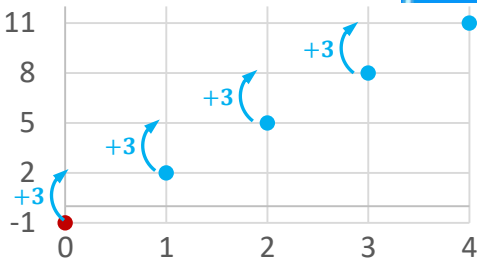
## Linear sequences:

$n^{\text{th}}$  term of 2, 5, 8, 11 ...

$n$	0	1	2	3	4
Term	-1	2	5	8	11
$3n$	0	3	6	9	12

$$n^{\text{th}} \text{ term} = 3n - 1 = -1 + 3n$$

$$y = 3n - 1$$

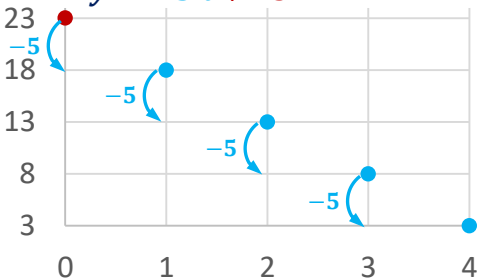


$n^{\text{th}}$  term of 18, 13, 8, 3 ...

$n$	0	1	2	3	4
Term	23	18	13	8	3
$-5n$	0	-5	-10	-15	-20

$$n^{\text{th}} \text{ term} = -5n + 23 = 23 - 5n$$

$$y = -5n + 23$$





# Algebra and graphs C2.10



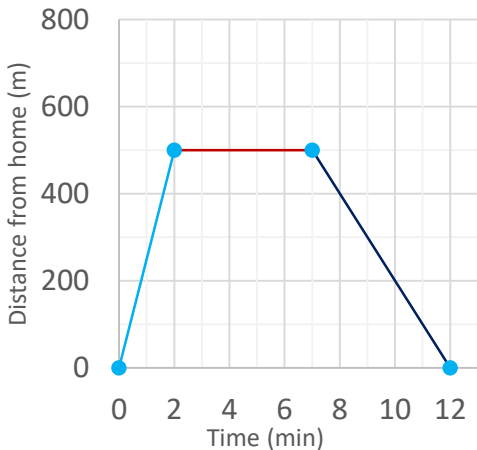
## Travel graphs:

**Speed 1:**  $500\text{m}/2\text{min} = 250\text{m}/\text{min}$

**Speed 2:**  $0\text{m}/5\text{min}$  (*stationary*)

**Speed 3:**  $500\text{m}/5\text{min} = 100\text{m}/\text{min}$

**Total speed:**  $\frac{1000\text{m}}{12\text{min}} = 83\frac{1}{3}\text{m}/\text{min}$

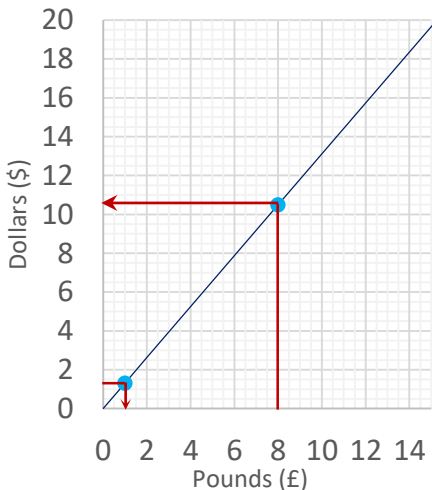


## Conversion graphs:

**Rate:** \$1.31 per £1

£1 = \$1.31

£8 = \$10.48



# Algebra and graphs C2.11



$$y = 3x - 1 = 3\left(x - \frac{1}{3}\right)$$

$x$	-4	-2	0	2	4
$y$	-13	-7	-1	5	11

$$y = 0 \text{ at } x = \frac{1}{3}$$

$$y = x^2 - x - 2 = (x - 2)(x + 1)$$

$x$	-4	-2	0	2	4
$y$	18	4	-2	0	10

$$y = 0 \text{ at } x = -1, x = 2$$

Estimate from graph:

$$x^2 - x - 2 = 3x - 1 \quad \{-4 \leq x \leq 4\}$$

$$x \approx -0.24, y \approx -1.7 \text{ at } \times$$

